

## Supplemental discussion

In the absence of lateral constraints, the critical force is given by the equation

$$f_c = \frac{A\kappa}{L^2},$$

where the prefactor  $A$  depends on the boundary conditions. If the ends are free to pivot,  $A = \pi^2$  ( $\approx 10$ ); if one end is clamped,  $A \approx 20$ ; if both ends are clamped,  $A = 4\pi^2$  ( $\approx 40$ ) (Landau and Lifshitz, 1986; Dogterom and Yurke, 1997).

In the constrained buckling equation, for a homogenous, incompressible, isotropic medium,  $\alpha$  is given by the equation

$$\alpha = \frac{4\pi G}{\ln(l/a)},$$

where  $l$  is a characteristic length scale of the buckling (i.e., the wavelength, which we observe to be on the order a few micrometers for microtubules) and  $a$  is a microscopic length of order for the rod radius, which is  $\sim 10$  nm for microtubules (Landau and Lifshitz, 1986). For microtubules, we thus have the following equation:

$$\alpha \approx \frac{4\pi G}{\ln(100)} \approx 2.7G.$$

To solve this equation, we look for solutions of mechanical equilibrium, for which

$$\frac{\delta E}{\delta u} = 0,$$

which implies  $\kappa u'''' + fu'' + \alpha u = 0$ . The solutions to this equation are of the form  $u \approx e^{ikx}$ . Oscillatory buckling solutions are given by purely real wavevector  $k$ . Thus, we must solve  $\kappa k^4 - fk^2 + \alpha = 0$ , for which the solution is the following equation:

$$k^2 = \frac{1}{2\kappa} \left( f \pm \sqrt{f^2 - 4\kappa\alpha} \right).$$

The solutions for  $k$  are real only for compressive forces that exceed a finite critical or threshold value of  $f_c = 2\sqrt{\kappa\alpha}$ . This is the analogue of the finite Euler buckling threshold for an isolated elastic rod. The corresponding buckling wavelength of the response is the following equation:

$$\lambda = 2\pi \left( \frac{\kappa}{\alpha} \right)^{1/4}.$$

It is important to note that for the same boundary conditions (pivoting or clamped), the prefactors in the expression for the critical buckling force are the same for both long- and short-wavelength buckling. Thus, for instance, when comparing the Euler buckling of a microtubule of length  $L$  with the case of buckling with a short wavelength  $\lambda$ , there is an

overall multiplicative increase of the buckling force (without constraints) given by the following factor:

$$4\left(\frac{L}{\lambda}\right)^2.$$

The factor of 4 comes from the fact that Euler buckling of a rod of length  $L$  corresponds to a half wavelength that is equal to  $L$ . In the final expression, including lateral constraints, there is an additional factor of 2, giving the following equation:

$$f_c = 8\pi^2 \frac{\kappa}{\lambda^2}.$$

This has a very simple interpretation in terms of the competition between bending and elastic deformation of the surrounding medium; in mechanical equilibrium, these are just balanced, meaning that the additional force coming from elastic constraint is equal to the bending contribution. This results in a simple doubling of the total force.

In this analysis, we have made no assumptions about whether the surrounding network physically adheres to the rod or not; transverse motion will, in any case, displace the surrounding network because of topological entanglements. However, adhesion of the rod to the network can set a decay length for longitudinal forces because a compressive force applied at one end of the rod will be transferred to the network if there are physical connections between the two. The extreme case of this is a no-slip condition, in which longitudinal displacement of the rod locally drags the network with it. In this case, we expect a force balance between the compression/stretching of the rod (without bending) and the elasticity of the surrounding network. The former is characterized by a Young's modulus,  $\mu \approx E_{MT}\pi a^2$ , where  $E_{MT}$  represents the elastic modulus of the tubulin making up the microtubule, which is expected (de Pablo et al., 2003) to be  $\sim 1$  GPa. A simple scaling analysis suggests that a no-slip condition leads to a decay length on the order of  $\sqrt{\mu/\alpha} \approx a\sqrt{E_{MT}/G}$ . A similar expression for the decay length is known from shear lag models of composite materials (Hull and Clyne, 1996). In the cell, microtubules are physically connected (and not simply sterically coupled) to the surrounding cytoskeletal network (Svitkina et al., 1995), and, thus, a no-slip condition should hold. We thus expect the short-wavelength buckling to decay away from the tips of compressively loaded microtubules at the cell edge; the scaling analysis suggests this should be  $\sim 10$   $\mu\text{m}$  for microtubules in an elastic network of shear modulus 1 kPa, which is consistent with our observations (Figs. 1 and 2). Moreover, in cytochalasin-treated cells, the decay length appeared to be significantly increased, as predicted from this scaling (unpublished data). In the macroscopic experiment, we also observed a decay in buckling amplitude (Fig. 6). By embedding fluorescent tracer particles, we confirmed that the gelatin physically sticks to the plastic rod, imposing a no-slip condition. Thus, we expect the amplitude will decay over a length of  $\sim 10$  cm, which is also consistent with our observations. We note that in the cell, the no-slip condition may be modified by dynamic cross-links between microtubules and actin, which could therefore significantly increase this distance of force transmission.

## References

- de Pablo, P., I.A.T. Schaap, F.C. MacKintosh, and C.F. Schmidt. 2003. Deformation and collapse of microtubules on the nanometer scale. *Phys. Rev. Lett.* 91:098101.
- Hull, D., and T.W. Clyne. 1996. *An Introduction to Composite Materials*. Cambridge University Press, Cambridge. 326 pp.